Vector Geometry and Linear Algebra MATH 1300

Course Syllabus

Welcome to Vector Geometry and Linear Algebra, MATH 1300. We hope you find this course to be interesting and challenging.

Contacting your Instructor

For information on contacting your instructor as well as other important information from your instructor see the Instructor Letter link in your course website.

Course Description

The University of Manitoba *Undergraduate Calendar* describes this course as follows:

> *An introduction to vectors, matrices, systems of linear equations and three-dimensional geometry. Not to be held for credit with MATH 1210, MATH 1211, MATH 1220, MATH 1310 (136.131), MATH 1301, or the former MATH 1680 (136.168). Prerequisite: a minimum grade of 60% in Pre-calculus Mathematics 40S or the former Mathematics 40S (300), or a grade of C or better in the Mathematical Skills course taught by Extended Education. NOTE: A minimum grade of 70% in Applied Mathematics 40S may be used as a prerequisite to this course.*

This course has been designed as an introduction to vector geometry and linear algebra for students who have successfully completed the required high school prerequisites. A reasonable knowledge of some of the fundamentals of high school mathematics such as: working with signed numbers, fractions, basic algebraic manipulations, geometry, and some trigonometry is assumed. Calculus is not required for this course. Accuracy in mathematical calculations is a necessity.

Because the topics of vector geometry and linear algebra have wide applications in other fields of mathematics, students who intend to do further courses in linear algebra, graph theory, calculus of functions of several variables, or differential equations are required to take this course as a prerequisite. In addition, since linear algebra is an indispensable tool in many other disciplines, this course is highly recommended for students planning to study statistics, chemistry, physics, biology, engineering, computer science, economics, sociology, psychology, and business mathematics.

This course will examine some of the elements of rigorous and formal mathematical thinking by developing the subject through the use of theorems and their proofs. The use of algorithmic methods to solve systems of linear equations and to find the inverse of a matrix will be utilized. [An algorithm is a step-by-step procedure to carry out an operation and hence is a procedure that is adaptable for use on a computer.]

The course begins with the study of methods for solving systems of linear equations using matrices and elementary row operations will be studied. Matrices themselves will then be considered as algebraic objects that can be added, subtracted, and multiplied. Inverse of matrices as well as their applications to solve systems will be studied.

The determinant function, which is an operation on square matrices, will be studied both for itself and for its applications. Some of its applications are: determining when a square matrix is invertible, calculating the volume of a parallelepiped and solving a system of n linear equations in n variables by Cramer's rule.

Vectors, first as geometric objects and then as algebraic objects will be studied. Some theorems from Euclidean geometry are proved using vectors. Vectors are then used to study lines and planes in three-dimensional space.

Linear transformations, which is a function with inputs one vector and outputs another vector are then studied. Various types of linear transformation involving geometric properties are discussed as well as establishing the connection between these functions and matrix operation learned earlier in the course.

Finally the course ends with a discussion of eigenvalues and eigenvectors which have wide ranging applications, some of which will be discussed in the course, including diagonalization and find powers of square matrices.

What is mathematics?

The word mathematics is derived from the Greek word *mathematike,* meaning scientific knowledge. One dictionary gives the following definition for the word mathematics. Mathematics is "the systematic treatment of magnitude, relationships between figures and forms, and relations between quantities expressed symbolically" (Random House Dictionary of the English Language, 1966). Mathematics can be subdivided into many branches such as algebra, geometry, trigonometry, calculus, probability, statistics, etc. In this course, you will be studying the *vector geometry* and *linear algebra* branches.

Nature of the mathematical method

The development of each branch of mathematics follows a common pattern. The development is similar to the construction of a house. To start with there is an agreed upon terminology for naming the objects being studied and an agreed upon set of properties relating the objects being studied. [This is the basic foundation upon which the structure rests.] Reasoning is then used to deduce properties and new terminology is introduced, as needed, to continue the development of the subject. [This part is the construction of the structure.] For this course we assume as a basic foundation the material normally covered in the high school curriculum. You must be familiar with the various properties of numbers (such as 2 + 3 = 3 + 2), geometry (both Euclidean and analytic), and trigonometry.

Course Goals

Upon completion of this course you should be able to a variety of topics. This is including, but not limited to:

- recognize a system of linear equations in two or three variables as a set of lines or planes in two or three dimensional space;
- recognize the solution set of a system of linear equations in two or three variables as the set of all points in common to all lines or planes in the system;

- add, subtract, and multiply matrices;
- find the inverse of a square matrix when it exists;
- evaluate the determinant of a square matrix;
- use the determinant to determine whether or not a square matrix has an inverse;
- use Cramer's rule to solve a system of linear equations;
- understand the concept of a vector both as a geometric object and an algebraic object;
- add and subtract vectors geometrically and algebraically;
- multiply a vector by a scalar;
- calculate the dot product and the cross product (when defined) of two vectors;
- find vector equations of lines and planes;
- use the dot product to find the angle between two lines and also between a pair of planes;
- find an equation of the line of intersection of two planes;
- find the point of intersection of a line and a plane;
- determine whether two lines in three dimensional space intersect or do not intersect each other;
- determine whether a transformation is linear;
- find the standard matrix for any linear transformation;
- know the standard matrices for diations, contractions, rotations and reflections;
- be able to compute the eigenspaces for a matrix;
- be able to take the power of a matrix and do some applications;

Course Materials

No additional textbook are required for this course other than the course material on your course outline. Other references include

- Selected Chapters from the book "Elementary Linear Algebra", by Howard Anton (11th edition). This text is require for the in class version of the course. It is sold through the book store.
- Most elementary linear algebra text are acceptable. Some can be found in libraries under the call letters QA 184.
- An open text found at **http://net276.math.umanitoba.ca/kuttler.pdf [http://net276.math.umanitoba.ca/kuttler.pdf]**

http://www.math.odu.edu/~bogacki/lat/ [http://www.math.odu.edu/%7Ebogacki/lat/]

http://www.mathpropress.com/glossary/glossary.html [http://www.mathpropress.com/glossary/glossary.html]

Course Overview

The course content is divided into seven units.

Unit 1 Systems of linear equations

Systems of linear equations involving two variables are represented geometrically as lines in the plane. Systems of linear equations in three variables are represented geometrically as planes in three-dimensional space. Solutions of systems of linear equations are sets of points that satisfy all equations of the system. Systems of linear equations may have no solution, exactly one solution, or infinitely many solutions. To solve such systems, matrices are introduced, and a set of elementary row operations on these matrices are performed. One such method is called the Gauss-Jordan elimination method. Another procedure, called Gaussian elimination with back substitution, is introduced as an alternative method for solving a system of linear equations. Systems of linear homogeneous equations are also considered.

Unit 2 Matrices

Matrices, which were first introduced in unit 1 to solve a system of linear equations, are now considered as algebraic objects. Operations of addition, subtraction, and multiplication are defined for matrices. The existence of the inverse of a square matrix is considered, and a method is developed for calculating the inverse of a square matrix whenever the inverse exists. The method for calculating the inverse employs the elementary row operations introduced in unit 3.

Unit 3 Determinants

The definition of the determinant of a square matrix is introduced. The calculation of the determinant by cofactor expansion as well as by using elementary row operations is undertaken. Properties of the determinant function are studied. The determinant is used to determine if a square matrix has an inverse and to solve some systems of linear equations by Cramer’s rule.

Unit 4 Vector geometry

Vectors are first introduced as geometric objects, namely, as directed line segments or arrows. Operations of addition, subtraction, and multiplication by a scalar (real number) are introduced for these directed line segments. Rectangular Cartesian coordinate systems are introduced and used to give an algebraic representation for the vectors. The operations of a dot product and a cross product are introduced. Some theorems from Euclidean geometry are proved using vector methods.

Unit 5 Planes and lines in $\mathbb{R}\,3$

Both vector and nonvector form equations for lines and planes in three-dimensional space are introduced. Vector methods are used to calculate angles between lines and planes as well as to find the distance between geometric objects, such as the distance between a point and a line and the distance between a point and a plane.. Equations of lines and planes are used to find the points of intersection of lines and planes with each other.

Unit 6 Linear Transformations

The definition and examples of linear transformations are introduced as well as their connection to matrices. Discussion of the algebraic properties of these transformations are discussed as well as composing multiple linear transformations.

Unit 7 Eigenvalues and Eigenvectors

Eigenvalues and Eigenvectors are defined and method for calculation are discussed. Further, applications to these objects are discussed including diagonalization and powers matrices. Further applications are also mentioned.

Evaluation and Grading

Students are required to obtain a minimum grades of 40% on the final exam in order to pass the course regardless of term work grades.

There are four assignments, a midterm and a final examination.

- Assignment 1 worth 2.5%
- Assignment 2 worth 2.5%
- Assignment 3 worth 2.5%
- Assignment 4 worth 2.5%
- Midterm worth 30%
- Final Exam worth 60%

Assignments

There are four assignments due throughout the term. Your instructor will determine what will be included on each assignment as well as the due dates for each assignment.

If you are unable to submit an assignment on time, contact your instructor well in advance of the due date, for we cannot guarantee that the instructor will accept late assignments.

Final Examination

The final exam will be written at the University of Manitoba (UM), Fort Garry campus or at an approved off-campus location. **Students needing to write at an off-campus location must declare a location by the specified deadline date** (see off-campus declaration and policy under Student Resources on course homepage). **Students writing at the UM Fort Garry campus do not need to declare an exam location.**

The Registrar's Office is responsible for the final exam schedule which is available approximately one month after the start of the course.

The final examination will be two hours in length and will cover material from all six units in the course. The majority of the questions on the final exam will be computational in nature and similar to the questions in the notes. You will not be required to reproduce proofs of theorems on the final exam, but you may be asked to solve a problem whose solution requires an argument similar to that given in the proof of some theorem(s).

When answering examination questions you should show all your work as part marks may be assigned for an incomplete or incorrect answer. **Calculators will not be allowed during the writing of the final exam**

Grading Scale

As per mathematics department policy, letter grade cutoffs are determined at the end of the course and are not released to students.

A word of caution about the assignments and the final examination

Some students find that they do very well on the assignments, but they do not do nearly as well on the final examination. While your grades on the assignments will give you some idea of how well you are mastering the material, they may not indicate how well you will do on the examination, because the examination is written under very different circumstances. Because the assignments are open book, they do not require the amount of memorization that a closed-book examination re quires nor are they limited to a specific time period. Some students have told us that, based on the high marks they received on the assignments, they were overconfident and underestimated the time and effort needed to prepare for the final examination.

Please keep all this in mind as you prepare for the examination. If your course has a sample exam or practice questions, use them to practice for the examination by setting a time limit and not having any books available. Pay careful attention to the description of the type of questions that will be on your final examination. Preparing for multiple choice questions involves a different type of studying than preparing for essay questions. Don't underestimate the stress involved in writing a time-limited examination.

Study Aids

If you have not previously studied vector geometry or linear algebra, you will be introduced to many new concepts. New terms and new operations will be defined and their properties will be studied. To aid you in your study we suggest two projects that will help you in your understanding of the material.

- Construct a glossary. As each new term is introduced in the course notes, add the term to your glossary along with its accompanying definition. Keep this glossary handy for quick reference when

needed.

- Construct a list of theorems. As each new theorem is introduced in the course notes, add the theorem to your list. Keep this list handy for quick reference.

Plagiarism, Cheating and Exam Impersonation

You should acquaint yourself with the University's policy on plagiarism, cheating, and examination impersonation as detailed in the General Academic Regulations and Policy section of the University of Manitoba Undergraduate Calendar. Note: These policies are also located in your Distance and Online Education Student Handbook or you may refer to Student Affairs at **http://www.umanitoba.ca/student [http://www.umanitoba.ca/student]**

Distance and Online Education (DE) Student Resources

In your course website there are links for the following:

- Contacting Distance and Online Education Staff
- Distance and Online Student Handbook
- Distance and Online Education Website

Acknowledgments

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