Introduction to Calculus
MATH 1500

Course Syllabus

Course description

The *University of Manitoba Undergraduate Calendar* describes this course as follows:

> Differentiation and integration of elementary functions, with applications to maxima and minima, rates of change, area, and volume. Not to be held with MATH 1501, MATH 1510 (or 136.151), MATH 1520 (or 136.152), MATH 1530 (or 136.153), MATH 1680 (or 136.168), MATH 1690 (or 136.169), or 006.125, 010.115, 013.128, 013.129, 013.139, or 013.159). Prerequisite: a minimum grade of 60 per cent in Pre-calculus Mathematics 40S or the former Mathematics 40S (300), or a grade of "C" or better in either MATH 1000 (or 136.100) or the Mathematical Skills course taught by the Continuing Education Division.

Calculus is the basis for mathematical analysis and has been at the center of mathematics since its introduction in the seventeenth century. Although a fairly old and classical subject, it is very important, with a wide array of applications. It is an indispensable tool in virtually all sciences–it is used for analyzing models of changing phenomena in the world, from statistics of population fluctuation, through data management, to space travel.

The core of calculus, and hence analysis, is the concept of limit and, subsequently, derivatives and integrals of a function. Indeed, these three are the main foci of this course, and with them we enter the world of the infinite and infinitesimal. They allow us to take a snapshot of an object at one fixed instant, or to show that, say, the area between the hyperbola $y = \frac{1}{x^2}$ and the x-axis, from $x = 1$ to infinity, is exactly 1.

The bulk of this course is devoted to the definition, properties, and applications of these concepts.

Contacting your instructor

For information on contacting your instructor as well as other important information from your instructor see the Instructor Letter link in your course website.

Course objectives

The main objective of this course is the introduction of the concepts of **limit** and, **derivatives**. These two cover at least three quarters of the course that may be subdivided into three main subtopics:

a. definitions (of limit and derivative);
b. properties, and
c. applications.

You should try hard to understand (a), manipulate (b), and appreciate (c).

Along the way you will deal with many important functions, notably trigonometric, exponential, and logarithmic functions. You should be able to apply the theories we establish for these functions.

The last, but certainly not the least main objective of this course, is a theorem that may very well be characterized by some as the main theorem of mathematics: you will learn about the Fundamental Theorem of Calculus.

Course materials

The following required materials are available for purchase from the University of Manitoba Bookstore. Please order your materials immediately, if you have not already done so. See your *Distance and Online Education Student Handbook* for instructions on how to order your materials.

Required textbook

Stewart, James. (2013). *Single Variable Calculus: Early Transcendentals,* Metric International Version (Packaged with students Solutions Manual). 7th edition. Nelson Education.

The material we cover is standard and may be found in almost every calculus textbook. The content of the textbook is integrated into the course materials. If you need a review of basic calculus, then you should browse through the earlier chapters of the textbook.

Course content

There are three main topics in this course.

Limits

First we shall define the *limit* of a function, followed by examples: some specific limits will be computed using the definition. In this way an initial class of relatively simple functions will be built. After introducing various laws and properties of limits we shall enlarge our class of functions to which we can apply limits. Finally, using *limit* we introduce the important concept of **continuity**.

Derivatives

This is the core of the course, indeed, its largest part. Derivatives are specific limits that measure rates of change. We introduce them through tangents and velocity. The theory is then slowly built by listing and using the rules of differentiation (product, quotient, chain rule), and by introducing and analyzing various types of functions (implicitly defined functions, trigonometric functions, exponential functions, etc.). We apply this to problems dealing with rates of change of functions.

In the second part, we expand and apply our theory. Note the Mean Value Theorem: it is one of the main ingredients of the theory that is built into this course. We use the derivatives to analyze the graphs of functions and deal with specific optimisation problems.

Integrals

The indefinite integral is essentially an antiderivative of a function: given a function f, its antiderivative is a function g, such that the derivative of g is f. The link between indefinite integrals and definite integrals is established by the Fundamental Theorem of Calculus. The last part of the course deals with this important theorem.

Required Proofs

The following proofs are required knowledge for this course. The proofs themselves can be found in the textbook under the following sections.

Proof	Section
differentiable $\rightarrow$ continuous	2.8
$(cf)' = cf'$	3.1
$(f+g)' = f' + g'$	3.1
$(fg)' = f'g + g'f$	3.2
$(\sin x)' = \cos x$	3.3
$f' = 0$ on $I \rightarrow f$ is constant on I	4.2
$f' > 0$ on $I \rightarrow f$ is increasing on I	4.3
$f' < 0$ on $I \rightarrow f$ is decreasing on I	4.3

Course outline

These sections will be assigned from Stewart, *Single Variable Calculus*.

1.1. Four Ways to Represent a Function: Representing functions.

1.3. New Functions from Old Functions: Properties of functions.

1.5. Exponential Functions: Properties of exponential functions.

1.6. Inverse Functions and Logarithms: Logarithms are inverse functions to exponential functions.

2.1. The Tangent and Velocity Problem: Introduction to limits.

2.2. The Limit of a Function: Definition.

2.3. Calculating Limits Using the Limit laws: Various properties of limits, including the Squeeze Theorem.

2.5. Continuity: Definition, then theorems establishing continuity of some types of functions. As an application, we have the Intermediate Value Theorem.

2.6. Limits at Infinity; Horizontal Asymptotes: New types of limits are introduced and then applied to certain graphs of functions.

2.7. Derivatives and Rates of Change: This is an introduction to differentiation, for the limits considered in this section are in fact derivatives.

2.8. The Derivative as a Function: Examples followed by a theorem (differentiable implies continuous).

3.1. Derivatives of Polynomial and Exponential Functions: Derivatives of some special classes of functions.

3.2. The Product and The Quotient Rules: Some basic properties of differentiation.

3.3. Derivatives of Trigonometric Functions.

3.4. Chain Rule: An important property of derivatives.

3.5. Implicit Differentiation: Differentiation of implicitly given functions.

3.6. Derivatives of Logarithmic Functions: Differentiation of Logarithmic Functions and applications.

3.9. Related Rates: Applying derivatives.

4.1. Maximum and Minimum Values: Investigating the extreme of functions by means of derivatives.

4.2. The Mean Value Theorem: Important theorem, a basis for other results, notably the Fundamental Theorem of Calculus.

4.3. How Derivatives Affect the Shape of a Graph: Investigating functions through their derivatives.

4.5. Summary of Curving Sketching: Analyzing functions by means of derivatives and then sketching their graphs.

4.7. Optimization Problems: Another application of derivatives.

4.9. Antiderivatives: Introducing indefinite integrals.

5.1. Areas and Distances: A Geometrical Introduction of Definite Integrals.

5.2. The Definite Integral: Definition and some properties.

5.3. Fundamental Theorem of Calculus: The link between definite integrals and (anti) derivatives.

5.4. Indefinite Integrals: An introduction to a common notation for anti-derivatives and a summary of anti-derivatives.

A note on the organization of the units

At the beginning of each unit, which covers selected topics in the course, we state the learning objectives, and then we list the topics from the textbook that you should read. This part ends with a reference to the sets of exercises that follow sections in the textbook; typically, you will be asked to do as many of them as you need. The problems needing graphing software are optional.

It is important to realize that reading mathematics is very different from reading novels: to read mathematics means primarily to **do**. This includes absorbing and understanding the definitions and statements, then studying and solving the examples. Active learning also includes asking (and, hopefully answering) as many reasonable questions as you can pose, thus opening up other perspectives. Hence, when you are asked to read parts of the textbook, you should remember to interpret the word "read" appropriately.

The main part of each unit consists of examples, comments, solutions, and somewhat unsuccessful attempts to solve the problems where we point out some typical errors. Almost all of the incorrect solutions are authentic: they have been gathered through many years of marking assignments, tests, and exams. We introduce a fictional character, whom we call Mark Markless, and to whom we mercilessly attribute almost all of the errors. Some of the correct solutions are illustrated with pictures or graphs. In some instances these are parts of the solution; in others, they are just a visualization of the solution. To avoid misunderstanding, the end of the solution is indicated by $\Diamond$.

Each unit ends with a self test; almost all of the questions are taken from old exams or assignments.

Evaluation and grading

Assignments

The solutions will be posted on the course website once all assignments have been graded.

Each lesson corresponds to approximately two week's work in the regular session of the University day course. You are encouraged to organize your own pacing for the course in order to meet all due dates.

Assignment due dates

Assignment due dates are designed to help you pace your coursework, as well as to enable you to receive valuable feedback on completed assignments for subsequent termwork and the exam. Deadlines for the submission of assignments can be found in the "Assignment Due Dates" document in your course.

Please be sure to submit assignments on time. We cannot guarantee an instructor will accept late assignments. If you do find that you are not able to complete an assignment on time, please be sure to contact your instructor well in advance of the due date in case your request is denied.

Occasionally students, under extenuating circumstances, (e.g., medical, military assignments, family difficulties, unexpected business travel), are unable to complete all assignments by the final deadline. Formal time extensions (beyond the date of close of term) must be directed to the instructor, and result in the assignment of a temporary grade of "Incomplete." Students must request these extensions in writing by the last assignment due date and in addition to obtaining the instructor's approval, must meet the following criteria:

> Students should be able do demonstrate that they have completed at least 50% of their half-course course term work to be eligible for the Incomplete grade classification.

Complete the assignment for the unit and submit it to the designated UMLearn Dropbox. For instructions on submitting your assignment, click on the "How to submit" link in the Instruction for Assignment widget in UMLearn.

Due dates

Assignment	Sept. – Dec.	Jan. – Apr.	May – Aug.
1	Sept. 30	Jan. 23	May 22
2	Oct. 19	Feb. 6	June 5
3	Nov. 14	Feb. 20	June 19
4	Dec. 1	Mar. 6	July 3

Midterm Examination

A one-hour, midterm examination will be held on Friday, October 28th from 5:30-6:30pm.

Students who live in Winnipeg are required to write the midterm examination at the UM Fort Garry Campus. A location on campus will be posted in UM Learn closer to the date.

Students who live outside of Winnipeg are required to take the midterm examination using a Virtual Proctoring Service. The online midterm examination will be posted on the course website.

Remote proctored exams cannot be written in a public venue. Students must write their exams in a private room that contains all equipment needed for the exam (printer, scanner, webcam, etc.).

Detailed information regarding the invigilation process will be available in UM Learn closer to the examination date.

Final examination

Students are required to obtain a minimum grade of 40% in the final exam in order to pass the course regardless of term work grades.

The final examination will be written at the University of Manitoba (UM), Fort Garry campus or at an approved off-campus location. **Students needing to write at an off-campus location must declare a location by the specified deadline date** (see off-campus declaration and policy under Student Resources on course homepage). **Students writing at the UM Fort Garry campus do not need to declare an exam location.**

The Registrar's Office is responsible for the final exam schedule which is available approximately one month after the start of the course.

The examination will consist of approximately 10–11 long answer problems, similar in nature to those in the homework assignments. A sample final exam will be available on the course site 1 month prior to the exam.

Note: The general policy of the mathematics department does not allow using any aids, including calculators.

A word of caution about the assignments and the final examination

Some students find that they do very well on the assignments, but they do not do nearly as well on the final examination. While your grades on the assignments will give you some idea of how well you are mastering the material, they may not indicate how well you will do on the examination, because the examination is written under very different circumstances. Because the assignments are open book, they do not require the amount of memorization that a closed-book examination requires nor are they limited to a specific time period. Some students have told us that, based on the high marks they received on the assignments, they were overconfident and underestimated the time and effort needed to prepare for the final examination.

Please keep all this in mind as you prepare for the examination. If your course has a sample exam or practice questions, use them to practice for the examination by setting a time limit and not having any books available. Pay careful attention to the description of the type of questions that will be on your final examination. Preparing for multiple choice questions involves a different type of studying than preparing for essay questions. Do not underestimate the stress involved in writing a time-limited examination.

Grading percentages

Assignments	10%
Midterm Examination	30%
Final Examination	60%
Total	100%

Note: Minimum grade of 40% required on the final exam in order to pass the course regardless of term work grades.

Please note: All final grades are subject to departmental review.

Guidelines for preparing assignments

Solutions to the assigned text must be carefully prepared, justifying each step. We recommend the following procedure in solving the assignment problems. **Note: You are not required to type your assignments.**

1. Read the assigned text as indicated in every unit. This gives you the base for understanding new topics and helps you develop necessary technique for problem solving.
2. Read the course materials, paying special attention to sections where problems together with the complete solutions are presented. We strongly encourage you to attempt to solve the problems from the course manual before you look at their solutions.
3. Try to solve the Practice problems provided in the course materials. The answers to these problems are found at the end of each unit.
4. Solve the Assignment problems. If you are having difficulty, review the text and course materials for solution procedures. If difficulties persist, contact your instructor for help.

Plagiarism, cheating, and examination impersonation

You should acquaint yourself with the University's policy on plagiarism, cheating, and examination impersonation as detailed in the General Academic Regulations and Policy section of the University of Manitoba *Undergraduate Calendar*. Note: These policies are also located in your *Distance and Online Education Student Handbook* or you may refer to Student Affairs at http://www.umanitoba.ca/student.

Distance and Online Education (DE) Student Resources

In your course website there are links for the following:

- Contact Distance and Online Education Staff
- Distance and Online Student Handbook
- Distance and Online Education Website

Acknowledgments

Content specialist:
Sasho Kalajdzievski, Ph.D.
Department of Mathematics
University of Manitoba

Sasho Kalajdzievski was born in Skopje, Macedonia, where he received his bachelor's and master's degrees in mathematics. He completed his doctoral studies at the University of Toronto, specializing in algebra/group theory, which remains his primary research field. He has taught numerous courses in mathematics in Skopje, Toronto and Winnipeg.

Editor:
James B. Hartman, Ph.D.
Distance and Online Education
University of Manitoba

Desktop publisher:
Lorna Allard
Distance and Online Education
University of Manitoba

University of Manitoba, Distance and Online Education